INTERNATIONAL JOURNAL OF ADVANCED RESEARCH IN ENGINEERING, SCIENCE & MANAGEMENT

# EXPLICIT AND RECURSIVE FORMULAE FOR THE CLASS OF GENERALIZED FIBONACCI SEQUENCE

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***Abstract:*** ***Fibonacci sequence*** $\{F_n\}$ ***is defined by the recurrence relation*** $F_n = F_{n-1} + F_{n-2}$ ***for*** $n \geq 2$ ***with initial condition*** $F_0 = 0, F_1 = 1.$ ***This sequence has been generalized in many ways, some by preserving the initial conditions, and others by preserving the recurrence relation. One of the generalizations of the Fibonacci sequence is the class of sequences*** $\left\{F_n^{(a,\,b)}\right\}$ ***generated by the recurrence relation***

$$F_n^{(a,\,b)} = \begin{cases} aF_{n-1}^{(a,\,b)} + F_{n-2}^{(a,\,b)} \ ; \ when\ n\ is\ odd \\ bF_{n-1}^{(a,\,b)} + F_{n-2}^{(a,\,b)} \ ; \ when\ n\ is\ even \end{cases} (n \geq 2)$$

***with initial condition*** $F_0^{(a,\,b)} = 0,\ F_1^{(a,\,b)} = 1$ ***and a, b are positive integers.***

***In this paper we express*** $F_n^{(a,\,b)}$ ***in simple explicit form and use it to derive the recursive formula for*** $F_n^{(a,\,b)}$ ***to compute its successor and predecessor. We also compute the value of 'generalized golden proportion' for this sequence.***

***Keywords:*** **Binet formula, Fibonacci sequence, Generalized Fibonacci sequence, Recursive formula.**

## 1. INTRODUCTION:

Many papers concerning a variety of generalizations of Fibonacci sequence have appeared in recent years [1, 2]. Readers can refer the book [3] for some basic background information and more details on the topic. Edson, Yayenie [4] generalized this sequence in to new class of generalized sequence which depends on two real parameters used in a recurrence relation.

**Definition**: For any two positive numbers *a* and *b*, the generalized Fibonacci sequence $\left\{F_n^{(a,\,b)}\right\}_{n=0}^{\infty}$ is defined recursively by $F_0^{(a,\,b)} = 0,\ F_1^{(a,\,b)} = 1$ and

$$F_n^{(a,\,b)} = \begin{cases} aF_{n-1}^{(a,\,b)} + F_{n-2}^{(a,\,b)} \ ; if\ n\ is\ odd \\ bF_{n-1}^{(a,\,b)} + F_{n-2}^{(a,\,b)} \ ; if\ n\ is\ even \end{cases} \text{where } n \geq 2.$$

Clearly $F_n^{(1,\,1)} = F_n$, a Fibonacci sequence. In [4], authors proved the extension of Binet's formula for this generalized Fibonacci sequence.

 *ISSN : 2394-1766* 

**Theorem** [4]: The terms of the generalized Fibonacci sequence $\left\{F_n^{(a,\,b)}\right\}_{n=0}^{\infty}$ are given by

$$F_n^{(a,\,b)} = \frac{b^{1-\xi(n)}}{(ab)^{\lfloor n/2 \rfloor}}\left(\frac{\alpha^n - \beta^n}{\alpha - \beta}\right)$$

where $\alpha = \left(\frac{ab+\sqrt{a^2b^2+4ab}}{2}\right)$, $\beta = \left(\frac{ab-\sqrt{a^2b^2+4ab}}{2}\right)$ and $\xi(n) := n - 2\left\lfloor\frac{n}{2}\right\rfloor$.

**Remark:** We shall use this in its equivalent form as

$$F_n^{(a,\,b)} = \frac{b^{1-\chi(n)}}{(ab)^{\lfloor n/2 \rfloor}}\left(\frac{\alpha^n-\beta^n}{\alpha-\beta}\right) \qquad (1)$$

with $\alpha = \left(\frac{ab+\sqrt{a^2b^2+4ab}}{2}\right)$, $\beta = \left(\frac{ab-\sqrt{a^2b^2+4ab}}{2}\right)$ and $\chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even} \end{cases}$.

In this paper we express (1) in simple explicit form. We use it to derive the recursive formula for $F_n^{(a,\,b)}$ to compute its successor and predecessor.

## 2. A simple form for $F_n^{(a,\,b)}$:

Here we express $F_n^{(a,\,b)}$ in the simple explicit form. We prove the following result:

**Theorem 1:** $F_n^{(a,\,b)} = \left\lfloor \gamma\alpha^n + \frac{1}{2}\right\rfloor$, where $= \frac{b^{1-\chi(n)}}{(ab)^{\lfloor n/2 \rfloor}(\alpha-\beta)}$.

**Proof:** By using (1) we have

$$\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| = |\gamma|\,|\beta^n| = \frac{b^{1-\chi(n)}}{(ab)^{\lfloor n/2 \rfloor}\sqrt{a^2b^2+4ab}}\left|\frac{ab-\sqrt{a^2b^2+4ab}}{2}\right|^n. \qquad (2)$$

If we consider $a = b = 1$ then it is easy to see that $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{1}{2}$.

We now consider the cases when $a \neq 1, b \neq 1$.

Case 1: Here we consider $ab \leq 4$. In this case we get $1 \leq \frac{4}{ab} \leq 2$. This gives $\left|1 - \sqrt{1+\frac{4}{ab}}\right| \leq \frac{3}{4}$. And hence

$$\left|1 - \sqrt{1+\frac{4}{ab}}\right|^n < 1.$$

Using this in (2) we get

$$\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{b^{1-\chi(n)}}{(ab)^{\lfloor n/2 \rfloor}\sqrt{a^2b^2+4ab}}\,\frac{(ab)^n}{2^n}. \qquad (3)$$

Now if $n$ is even, say $n = 2k$, then $\chi(n) = 0$. In this case

$$\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{b}{\sqrt{a^2b^2+4ab}}\,\frac{(ab)^k}{4^k} < \frac{b}{\sqrt{a^2b^2+4ab}}.$$

Since $ab \leq 4$ (and $a \geq 2$) we have $(a^2-4)b^2 + 4ab > 0$. This gives $\frac{b}{\sqrt{a^2b^2+4ab}} < \frac{1}{2}$.

Thus we have $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{1}{2}$.

Again if $n$ is odd, say $n = 2k+1$, we have $\chi(n) = 1$. Then from (3) we get

$$\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{ab\left(\frac{ab}{4}\right)^k}{2\sqrt{a^2b^2+4ab}} < \frac{ab}{2\sqrt{a^2b^2+4ab}} < \frac{1}{2}.$$

Thus for any value of *n* we have $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{1}{2}$, i.e. $\frac{-1}{2} < F_n^{(a,\,b)} - \gamma\alpha^n < \frac{1}{2}$.

This gives $F_n^{(a,\,b)} = \left[\gamma\alpha^n + \frac{1}{2}\right]$.

Case 2: In this case we consider $ab > 4$. Then we observe that $\left|1 - \sqrt{1 + \frac{4}{ab}}\right|^n < \left(\frac{2}{ab}\right)^n$.

Then from (2) we get $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| = \frac{\gamma(ab)^n}{2^n}\left|1 - \sqrt{1 + \frac{4}{ab}}\right|^n < \frac{\gamma(ab)^n}{2^n}\left(\frac{2}{ab}\right)^n$.

This gives $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{b^{1-\chi(n)}}{(ab)^{\lfloor n/2 \rfloor}\sqrt{a^2b^2+4ab}}$.

As above here also we consider the cases when *n* is even or odd. If $n = 2k$ then $\chi(n) = 0$. Thus

$$\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{b}{(ab)^k\sqrt{a^2b^2+4ab}}.$$

Now since $ab > 4$, we have $\frac{1}{\sqrt{a^2b^2+4ab}} < \frac{1}{4\sqrt{2}}$. Then $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{b}{(4)^{k-1}4\sqrt{2}(ab)} < \frac{1}{2}$.

Also if $n = 2k + 1$ then $\chi(n) = 1$. Thus by (2) we get $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{1}{4\sqrt{2}(4)^k} < \frac{1}{2}$.

Thus $\left|F_n^{(a,\,b)} - \gamma\alpha^n\right| < \frac{1}{2}$.

Hence in any case we get $F_n^{(a,\,b)} = \left\lfloor\gamma\alpha^n + \frac{1}{2}\right\rfloor$, as required.

## 3. Successor and predecessor for $F_n^{(a,\,b)}$:

Here first we derive the result for the value of $F_{n+1}^{(a,\,b)}$, when the value of $F_n^{(a,\,b)}$ is known.

**Theorem 2:** $F_{n+1}^{(a,\,b)} = \left\lfloor\frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} + \frac{1}{2}\right\rfloor$; $n \geq 2$ and $\chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even} \end{cases}$.

**Proof:** We write (1) as $F_n^{(a,\,b)} = \gamma(\alpha^n - \beta^n)$, where $\gamma = \frac{b^{1-\chi(n)}}{(ab)^{\lfloor n/2 \rfloor}(\alpha-\beta)}$.

If we consider $a = b = 1$ then for the case of classical Fibonacci sequence, Koshy [3] proved that $F_{n+1}^{(a,b)} = \left\lfloor\alpha F_n^{(a,b)} + \frac{1}{2}\right\rfloor$, which is analogues to our result.

Thus throughout we consider the case when $a \neq 1, b \neq 1$.

Now consider the case when n is even, say $n = 2k$.

$\therefore F_{2k}^{(a,\,b)} = \frac{b}{(ab)^{\lfloor 2k/2 \rfloor}}\left(\frac{\alpha^{2k}-\beta^{2k}}{\alpha-\beta}\right) = \frac{b}{(ab)^k}\left(\frac{\alpha^{2k}-\beta^{2k}}{\alpha-\beta}\right)$ and $F_{2k+1}^{(a,\,b)} = \frac{1}{(ab)^k}\left(\frac{\alpha^{2k+1}-\beta^{2k+1}}{\alpha-\beta}\right)$.

$$\begin{aligned} bF_{2k+1}^{(a,\,b)} - aF_{2k}^{(a,\,b)} &= \frac{1}{(ab)^k(\alpha-\beta)}\left(b\alpha^{2k+1} - b\beta^{2k+1} - b\alpha^{2k+1} + ba\beta^{2k}\right) \\ &= \frac{b\beta^{2k}(a-\beta)}{(ab)^k(\alpha-\beta)} \\ &= \frac{b}{(ab)^k}\left[\frac{ab-\sqrt{a^2b^2+4ab}}{2}\right]^{2k} = \frac{b(ab)^{2k}}{(ab)^k 2^{2k}}\left(1 - \sqrt{1 + {}^4/_{ab}}\right)^{2k}. \end{aligned} \tag{4}$$

Here also we consider the two cases.

Case 1: First consider $ab \le 4$. Then $1+\frac{4}{ab} > \left(1-\frac{1}{2ab}\right)^2$. This gives $\left|1-\sqrt{1+\frac{4}{ab}}\right|^{2k} < \frac{1}{2ab}$.

Thus (4) becomes $\left|bF_{2k+1}^{(a,b)} - aF_{2k}^{(a,b)}\right| < \left|\frac{b(ab)^{2k}}{(ab)^k 2^{2k}}\right| \frac{1}{2ab} = \left|\frac{b(ab)^k}{2^{2k}2ab}\right| = b\left(\frac{ab}{4}\right)^k \frac{1}{2ab}$.

Now since $\frac{ab}{4} < 1, \left|bF_{2k+1}^{(a,b)} - aF_{2k}^{(a,b)}\right| < \frac{1}{2a} < \frac{1}{2}$.

Thus $\left|bF_{2k+1}^{(a,b)} - aF_{2k}^{(a,b)}\right| < \frac{1}{2}$.

Case 2: Here we consider the case when $ab > 4$.

In this case we get $\left|1-\sqrt{1+\frac{4}{ab}}\right|^{2k} < \left|\frac{2}{ab}\right|^{2k}$. Using this in (4) we get

$$\left|bF_{2k+1}^{(a,b)} - aF_{2k}^{(a,b)}\right| = \left|\frac{b(ab)^{2k}}{(ab)^k 2^{2k}}\left(1-\sqrt{1+\frac{4}{ab}}\right)^{2k}\right| < \frac{b}{2^{2k}}(ab)^k\left(\frac{2}{ab}\right)^{2k} = \frac{b}{(ab)^k} = \frac{b}{ab(ab)^{k-1}}.$$

Since $ab > 4$, we get $\frac{1}{ab} < \frac{1}{4}$. Thus $\left(\frac{1}{ab}\right)^{k-1} < \frac{1}{4^{k-1}}$.

$\therefore \left|bF_{2k+1}^{(a,b)} - aF_{2k}^{(a,b)}\right| < \frac{1}{a(ab)^{k-1}} < \frac{1}{a4^{k-1}} < \frac{1}{2}$.

This gives $\left|bF_{2k+1}^{(a,b)} - aF_{2k}^{(a,b)}\right| < \frac{1}{2}$.

In any case this gives $F_{2k+1}^{(a,b)} < \frac{a}{b}F_{2k}^{(a,b)} + \frac{1}{2} < F_{2k+1}^{(a,b)} + 1$.

This finally gives $F_{2k+1}^{(a,b)} = \left\lfloor\frac{a}{b}F_{2k}^{(a,b)} + \frac{1}{2}\right\rfloor$. (5)

We next consider the case when $n$ is odd, say $n = 2k+1$.

$\therefore F_{2k+1}^{(a,b)} = \frac{1}{(ab)^k}\left(\frac{\alpha^{2k+1}-\beta^{2k+1}}{\alpha-\beta}\right)$ and $F_{2k+2} = \frac{b\left(\alpha^{2k+2}-\beta^{2k+2}\right)}{(ab)^{k+1}(\alpha-\beta)}$. Thus we have

$$aF_{2k+2}^{(a,b)} - \alpha F_{2k+1}^{(a,b)} = \frac{ab\alpha^{2k+2} - ab\beta^{2k+2} - ab\alpha^{2k+2} + ab\alpha\beta^{2k+1}}{(ab)^{k+1}(\alpha-\beta)}$$

$$= \frac{ab\,\beta^{2k+1}(\alpha-\beta)}{(ab)^{k+1}(\alpha-\beta)}$$

$$= \frac{\beta^{2k+1}}{(ab)^k} = \frac{1}{ab^k}\left[\frac{ab-\sqrt{a^2b^2+4ab}}{2}\right]^{2k+1}$$

$$= \frac{(ab)^{k+1}}{2^{2k+1}}\left[1-\sqrt{1+\frac{4}{ab}}\right]^{2k+1} = \frac{ab}{2}\left(\frac{ab}{4}\right)^k\left[1-\sqrt{1+\frac{4}{ab}}\right]^{2k+1}.$$

Case 1: First consider $ab \le 4$. Then$1+\frac{4}{ab} > \left(1-\frac{1}{2ab}\right)^2$.

This gives $\left|1-\sqrt{1+\frac{4}{ab}}\right|^{2k+1} < \frac{1}{2ab}$.

$\therefore \left|aF_{2k+2}^{(a,b)} - \alpha F_{2k+1}^{(a,b)}\right| < \left|\frac{ab}{2}\left(\frac{ab}{4}\right)^k \times \frac{1}{2ab}\right| < \frac{1}{4} < \frac{1}{2}$.

Case 2: Here we consider the case when $ab > 4$. Then $\left|1-\sqrt{1+\frac{4}{ab}}\right|^{2k+1} < \left|\frac{2}{ab}\right|^{2k+1}$.

Thus,

$$\left|aF_{2k+2}^{(a,\,b)} - \alpha F_{2k+1}^{(a,\,b)}\right| < \frac{(ab)^{k+1}}{2^{2k+1}}\left(\frac{2}{ab}\right)^{2k+1} = \frac{1}{(ab)^k}.$$

Since $ab > 4$ we have $\left(\frac{1}{ab}\right)^k < \frac{1}{4^k}$. This gives $\left|aF_{2k+2}^{(a,\,b)} - \alpha F_{2k+1}^{(a,\,b)}\right| < \frac{1}{4^k} < \frac{1}{2}$.

Thus $\left|F_{2k+2}^{(a,\,b)} - \frac{\alpha}{a}F_{2k+1}^{(a,\,b)}\right| < \frac{1}{2a} < \frac{1}{2}$.

$\therefore F_{2k+2}^{(a,\,b)} < \frac{\alpha}{a}F_{2k+1}^{(a,\,b)} + \frac{1}{2} < F_{2k+2}^{(a,\,b)} + 1$, which finally gives

$$F_{2k+2}^{(a,\,b)} = \left\lfloor \frac{\alpha}{a}F_{2k+1}^{(a,\,b)} + \frac{1}{2}\right\rfloor. \tag{6}$$

Combining (5) and (6) we get

$F_{n+1}^{(a,\,b)} = \left\lfloor \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} + \frac{1}{2}\right\rfloor$; n $\geq 2$, where $\chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even}\end{cases}$, as required.

We use this theorem to derive the 'golden proportion' for the sequence $\left\{F_n^{(a,\,b)}\right\}_{n=0}^{\infty}$.

**Corollary**: $\lim_{n\to\infty}\frac{F_{n+1}^{(a,\,b)}}{F_n^{(a,\,b)}} = \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}$; $n \geq 2$, where $\chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even}\end{cases}$.

Proof : By above theorem we have

$$F_{n+1}^{(a,\,b)} = \left\lfloor \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} + \frac{1}{2}\right\rfloor; \text{n} \geq 2, \text{ where } \chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even}\end{cases}.$$

This can be written as $F_{n+1}^{(a,\,b)} = \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} + \frac{1}{2} + \theta$, for some real number $\theta$.

$\therefore \frac{F_{n+1}^{(a,\,b)}}{F_n^{(a,\,b)}} = \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}} + \frac{1}{2F_n^{(a,\,b)}} + \frac{\theta}{F_n^{(a,\,b)}}$. This gives

$$\lim_{n\to\infty}\frac{F_{n+1}^{(a,\,b)}}{F_n^{(a,\,b)}} = \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}, \text{ where } \chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even}\end{cases}.$$

We conclude this paper by deriving the expression for $F_n^{(a,\,b)}$, when the value of $F_{n+1}^{(a,\,b)}$ is known.

**Theorem 3:** $F_n^{(a,\,b)} = \left\lfloor \frac{a^{\chi(n)}b^{1-\chi(n)}}{\alpha}\left(F_{n+1}^{(a,\,b)} + \frac{1}{2}\right)\right\rfloor$; $n \geq 2$, where $\chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even}\end{cases}$.

**Proof:** By above theorem we have

$$F_{n+1}^{(a,\,b)} = \left\lfloor \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} + \frac{1}{2}\right\rfloor; \text{n} \geq 2, \text{ where } \chi(n) = \begin{cases} 1; \text{if } n \text{ is odd} \\ 0; \text{if } n \text{ is even}\end{cases}.$$

$\therefore \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} - \frac{1}{2} < F_{n+1}^{(a,\,b)} \leq \frac{\alpha}{a^{\chi(n)}b^{1-\chi(n)}}F_n^{(a,\,b)} + \frac{1}{2}$.

This gives $\frac{a^{\chi(n)}b^{1-\chi(n)}}{\alpha}\left(F_{n+1}^{(a,\,b)} - \frac{1}{2}\right) \leq F_n^{(a,\,b)} < \frac{a^{\chi(n)}b^{1-\chi(n)}}{\alpha}\left(F_{n+1}^{(a,\,b)} + \frac{1}{2}\right)$.

Since $F_n^{(a,\,b)}$ is an integer, we get $F_n^{(a,\,b)} = \left\lfloor \frac{a^{\chi(n)}b^{1-\chi(n)}}{\alpha}\left(F_{n+1}^{(a,\,b)} + \frac{1}{2}\right)\right\rfloor$; $n \geq 2$.

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# TITLE OF PAPER

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***Abstract: Abstract should contain Proper Information about overall information of Research in 300 to 350 words. (It should be in Italic and in Times New Roman format with 1.5 vertical spacing between two lines and should be justified).***



***Keywords: Include at least 4 keywords or phrases and should be in Alphabetical Order.***


### INTRODUCTION

**The write up of Introduction should contain Technical data related to your research paper and should contain details of other researchers who have did same experimental work in their regions and does not contain any other extra details which are not related with your topic.**

**(Format of Introduction: It should be in Times New Roman format with 12 font size and vertical spacing should be 1.5 and sentences should be justified and contains Indent of 0.3" from left side).**

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*A.* Text Font of Entire Document

The entire document should be in Times New Roman. Other Type fonts must not be used. Other font types may be used if needed for special purposes.

Recommended font sizes are shown in Table 1.

TABLE I: - Font Sizes for Papers

| **Font Size** | **Appearance (in Time New Roman or Times)** | | |
|---|---|---|---|
| | **Regular** | **Bold** | **Italic** |
| 8 | table caption (in Small Caps), figure caption, reference item | | reference item (partial) |
| 9 | author email address (in Courier), cell in a table | abstract body | abstract heading (also in Bold) |
| 10 | level-1 heading (in Small Caps), paragraph | | level-2 heading, level-3 heading, author affiliation |
| 12 | author name | Bold | |
| 16 | Title | Bold | |

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No more than 3 levels of headings should be used. All headings must be in 12 pt font. Every word in a heading must be capitalized except for short minor words.

*1)* Level-1 Heading: A level-1 heading must be in Small Caps, centred and numbered using uppercase Roman numerals and should be bold. For example, see heading “Page Style” of this document.The two level-1 headings which must not be numbered are “Acknowledgment” and “References”.

*2)* Level-2 Heading: A level-2 heading must be in Italic, left-justified and numbered using an uppercase alphabetic letter followed by a period. For example, see heading “B. Section Headings” above.

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## CONCLUSION

- Conclusion Should Contain only Discussion in technical Directions
- It Should be with Given Bullets and in this pattern only

## ACKNOWLEDGMENT

The heading of the Acknowledgment and References must not be numbered.

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